

BARYON RESONANCES IN THE $1/N_C$ EXPANSION

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The $1/N_c$ expansion of QCD provides a valuable semiquantitative tool to study baryon scattering amplitudes and the short-lived baryon resonances embedded within them. A generalization of methods originally applied in chiral soliton models in the 1980's provides the key to deriving a rigorous $1/N_c$ expansion. One obtains model-independent relations among amplitudes that impose mass and width degeneracies among resonances of various quantum numbers. Phenomenological evidence confirms that patterns of resonant decay predicted by $1/N_c$ agree with data. One may extend the analysis to subleading orders in $1/N_c$, where again agreement with data is evident, in both meson-baryon scattering and pion photoproduction.

1. Introduction

About 150 of the 1100 pages in the 2004 *Review of Particle Properties*¹ catalogue measured properties of baryons; and of these, about 100 describe resonances unstable against strong decay, with lifetimes so short as to appear only as features in partial wave analyses. Such states have resisted a model-independent description for decades. To date there exists no convincing explanation for why QCD produces *any* baryon resonances, much less for their peculiar observed spectroscopy, mass spacings, and decay widths. Even the unambiguous existence of numerous resonances remains open to debate, as evidenced by the infamous 1- to 4-star classification system.¹

Baryon resonances are exceptionally difficult to study precisely because they *are* resonances rather than stable states. For example, treating baryon resonances as Hamiltonian eigenstates in quark potential models is questionable, because such models are strictly speaking valid only when vacuum $q\bar{q}$ pair production and annihilation is suppressed (to ensure a Hermitian Hamiltonian). It is just this mechanism, however, that provides the means by which baryon resonances occur in scattering from ground-state baryons.

Even so, one of the most natural descriptions of excited baryons in large

N_c remains an N_c valence quark picture. The inspiration for this choice is that the ground-state baryon multiplets ($J^P = \frac{1}{2}^+, \frac{3}{2}^+$ for $N_c=3$) neatly fill a single multiplet completely symmetric under combined spin-flavor symmetry [the SU(6) **56**, for 3 light flavors], so that one may suppose the ground state of N_c quarks is also completely spin-flavor symmetric. Indeed, the SU(6) spin-flavor symmetry for ground-state baryons is shown to become exact in the large N_c limit.² Then, in analogy to the nuclear shell model, excited states are formed by promoting a small number [$O(N_c^0)$] of quarks into orbitally or radially excited orbitals. For example, the generalization of the SU(6)×O(3) multiplet (**70**, 1^-) consists of $N_c - 1$ quarks in the ground state and one in a relative $\ell=1$ state. One may then analyze observables such as masses and axial-vector couplings by constructing a Hamiltonian whose terms possess definite transformation properties under the spin-flavor symmetry and are accompanied by known powers of N_c . By means of the Wigner-Eckart theorem, one then relates observables for different states in each multiplet. This approach has been extensively studied^{3,4,5,6,7,8} (see Ref. 9 for a short review), but it falls short in two important respects:

First, a Hamiltonian formalism is not entirely appropriate to unstable particles, since it refers to matrix elements between asymptotic external states. Indeed, a resonance is properly represented by a complex-valued pole in a scattering amplitude, its real and imaginary parts indicating mass and width, respectively. Moreover, a naive Hamiltonian does not recognize the essential nature of resonances as excitations of ground-state baryons.

Second, even a Hamiltonian constructed to respect the instability of the resonances would not necessarily give states in the simple quark-shell baryon multiplets as its eigenstates. Just as in the nuclear shell model, the possibility of *configuration mixing* suggests that the true eigenstates might consist of mixtures of states with 1, 2, or several excited quarks.

In contrast to quark potential models, chiral soliton models naturally accommodate baryon resonances as excitations resulting from scattering of mesons off ground-state baryons. Such models are consistent with the large N_c limit because the solitons are heavy, semiclassical objects compared to the mesons. As has been known for many years,¹⁰ a number of predictions following from the Skyrme and other chiral soliton models are independent of the details of the soliton structure, and may be interpreted as group-theoretical, model-independent large N_c results. Indeed, the equivalence of group-theoretical results for ground-state baryons in the Skyrme and quark models in the large N_c limit was demonstrated¹¹ long ago. Compared to quark models, chiral soliton models tend to fall short in providing detailed

spectroscopy and decay parameters for baryon resonances, particularly at higher energies. It is therefore gratifying that large N_c provides a point of reference where both pictures share common ground.

In the remainder of this talk I discuss how the chiral soliton *picture* (no specific model) may be used to study baryon resonances as well as the full scattering amplitudes in which they appear, and also its relation to the quark *picture* (again, no specific model). It summarizes a series of papers written in collaboration with Tom Cohen (and more recently our students),^{12,13,14,15,16,17,18} and updates an earlier version¹⁹ of this talk.

2. Amplitude Relations

In the mid-1980's a series of papers^{20,21,22,23,24} uncovered a number of linear relations between meson-baryon scattering amplitudes in chiral soliton models. The fundamentally group-theoretical nature of these results, as was pointed out, suggested consistency with the large N_c limit.

Standard N_c counting²⁵ shows that ground-state baryons have masses of $O(N_c^1)$, but meson-baryon scattering amplitudes are $O(N_c^0)$. Therefore, the characteristic resonant energy of excitation above the ground state and resonance widths are both generically expected to be $O(N_c^0)$. To say that two baryon resonances are degenerate to leading order in $1/N_c$ thus actually means equal masses at both the $O(N_c^1)$ and $O(N_c^0)$ levels.

A prototype of these linear relations was first derived in Ref. 22. For a ground-state (N or Δ) baryon of spin = isospin R scattering with a meson (indicated by the superscript) of relative orbital angular momentum L (and primes for analogous final-state quantum numbers) through a combined channel of isospin I and spin J , the full scattering amplitudes S may be expanded in terms of a smaller set of “reduced” scattering amplitudes s :

$$S_{LL'RR'IJ}^{\pi} = (-1)^{R'-R} \sqrt{[R][R']} \sum_K [K] \left\{ \begin{matrix} K & I & J \\ R' & L' & 1 \end{matrix} \right\} \left\{ \begin{matrix} K & I & J \\ R & L & 1 \end{matrix} \right\} s_{KL'L}^{\pi}, \quad (1)$$

$$S_{LRJ}^{\eta} = \sum_K \delta_{KL} \delta(LRJ) s_K^{\eta}, \quad (2)$$

where $[X] \equiv 2X+1$, and $\delta(j_1 j_2 j_3)$ indicates the angular momentum addition triangle rule. Both are consequences of a more general formula²⁶ involving $9j$ symbols that holds for mesons of arbitrary spin and isospin, which for brevity we do not reproduce here. The basic feature inherited from chiral soliton models is the quantum number K (*grand spin*) with $\mathbf{K} \equiv \mathbf{I} + \mathbf{J}$, conserved by the underlying hedgehog configuration, which breaks I and J

separately. The physical baryon state is a linear combination of K eigenstates that is an eigenstate of both I and J but no longer K . K is thus a good (albeit hidden) quantum number that labels the reduced amplitudes s . The dynamical content of relations such as Eqs. (1)–(2) lies in the s amplitudes, which are independent for each value of K allowed by $\delta(IJK)$.

In fact, K conservation turns out to be equivalent to the large N_c limit. The proof¹² begins with the observation that the leading-order (in $1/N_c$) t -channel exchanges have $I_t = J_t$,²⁷ which in turn is proved using large N_c consistency conditions²⁸—essentially, unitarity order-by-order in $1/N_c$ in meson-baryon scattering processes. However, (s -channel) K conservation was found—years earlier—to be equivalent to the (t -channel) $I_t = J_t$ rule,²⁴ due to the famous Biedenharn-Elliott sum rule,²⁹ an $SU(2)$ identity.

The significance of Eqs. (1)–(2) lies in the fact that there exist more full observable scattering amplitudes S than reduced amplitudes s . Therefore, one obtains a number of linear relations among the measured amplitudes holding at leading [$O(N_c^0)$] order. In particular, a resonant pole appearing in one of the physical amplitudes must appear in at least one reduced amplitude; but this same reduced amplitude contributes to a number of other physical amplitudes, implying a degeneracy between the masses and widths of resonances in several channels.¹² For example, we apply Eqs. (1)–(2) to negative-parity^a $I = \frac{1}{2}$, $J = \frac{1}{2}$ and $\frac{3}{2}$ states (called $N_{1/2}$, $N_{3/2}$) in Table 1. Noting that neither the orbital angular momenta L, L' nor the mesons π, η that comprise the asymptotic states can affect the compound state except by limiting available total quantum numbers (I, J, K), one concludes that a resonance in the $S_{11}^{\pi NN}$ channel ($K = 1$) implies a degenerate pole in $D_{13}^{\pi NN}$, because the latter contains a $K = 1$ amplitude. One thus obtains towers of degenerate negative-parity resonance multiplets labeled by K :

$$\begin{aligned} & N_{1/2}, \Delta_{3/2}, \dots \quad (K=0: s_0^\eta), \\ & N_{1/2}, \Delta_{1/2}, N_{3/2}, \Delta_{3/2}, \Delta_{5/2}, \dots \quad (K=1: s_{100}^\pi, s_{122}^\pi), \\ & \Delta_{1/2}, N_{3/2}, \Delta_{3/2}, N_{5/2}, \Delta_{5/2}, \Delta_{7/2}, \dots \quad (K=2: s_{222}^\pi, s_2^\eta). \end{aligned} \quad (3)$$

It is now fruitful to consider the quark-shell picture large N_c analogue of the first excited negative-parity multiplet [the $(\mathbf{70}, 1^-)$]. Just as for $N_c = 3$, there are two $N_{1/2}$ and two $N_{3/2}$ states. If one computes the masses to $O(N_c^0)$ for the entire multiplet in which these states appear, one finds only three distinct eigenvalues,^{6,12,30} which are labeled m_0 , m_1 , and m_2 and listed in Table 1. Upon examining an analogous table containing all the

^aParity enters by restricting allowed values of L, L' .¹³

Table 1. Application of Eqs. (1)–(2) to sample negative-parity channels.

State	Quark Model Mass	Partial Wave, K -Amplitudes	
$N_{1/2}$	m_0, m_1	$S_{11}^{\pi NN}$	$= s_{100}^{\pi}$
		$D_{11}^{\pi\Delta\Delta}$	$= s_{122}^{\pi}$
		$S_{11}^{\eta NN}$	$= s_0^{\eta}$
$N_{3/2}$	m_1, m_2	$D_{13}^{\pi NN}$	$= \frac{1}{2} (s_{122}^{\pi} + s_{222}^{\pi})$
		$D_{13}^{\pi N\Delta}$	$= \frac{1}{2} (s_{122}^{\pi} - s_{222}^{\pi})$
		$S_{13}^{\pi\Delta\Delta}$	$= s_{100}^{\pi}$
		$D_{13}^{\pi\Delta\Delta}$	$= \frac{1}{2} (s_{122}^{\pi} + s_{222}^{\pi})$
		$D_{13}^{\eta NN}$	$= s_2^{\eta}$

states in this multiplet,¹² one quickly concludes that exactly the required resonant poles are obtained if each K amplitude, $K=0, 1, 2$, contains precisely one pole, which is located at the value m_K . The lowest quark-shell multiplet of negative-parity excited baryons is found to be *compatible* with, *i.e.*, consist of a complete set of, multiplets classified by K . But the quark-shell masses are *real* Hamiltonian eigenvalues, and therefore present a result less general than that obtained from the K amplitude analysis.

One can prove¹³ this compatibility for all nonstrange baryon multiplets in the $SU(6) \times O(3)$ shell picture.^b It is important to note that compatibility does not imply $SU(6)$ is an exact symmetry at large N_c for resonances as it is for ground states.² Instead, it says that $SU(6) \times O(3)$ multiplets are *reducible* multiplets at large N_c . In the example given above, $m_{0,1,2}$ each lie only $O(N_c^0)$ above the ground state, but are separated by $O(N_c^0)$ intervals.

We emphasize that large N_c by itself does not mandate the existence of any resonances at all; rather, it merely tells us that if even one exists, it must be a member of a well-defined multiplet. Although the soliton and quark pictures both have well-defined large N_c limits, compatibility is a remarkable feature that combines them in a particularly elegant fashion.

3. Phenomenology

Confronting these formal large N_c results with experiment poses two significant challenges, both of which originate from neglecting $O(1/N_c)$ corrections. First, the lowest multiplet of nonstrange negative-parity states covers quite a small mass range (only 1535–1700 MeV), while $O(1/N_c)$ mass split-

^bStudies to extend these results to flavor $SU(3)$ are underway¹⁷; while the group theory is more complicated, it remains tractable.

tings can generically be as large as $O(100 \text{ MeV})$. Any claims that two such states are degenerate while two others are not must be carefully scrutinized. Second, the number of states in each multiplet increases with N_c , meaning that a number of large N_c states are spurious in $N_c = 3$ phenomenology. For example, for $N_c \geq 7$ the analogue of the **70** contains three $\Delta_{3/2}$ states, but only one $[\Delta(1700)]$ when $N_c = 3$. As N_c is tuned down from large values toward 3, the spurious states must decouple through the appearance of factors such as $(1 - 3/N_c)$, which in turn requires one to understand simultaneously leading and subleading terms in the $1/N_c$ expansion.

Nevertheless, it is possible to obtain testable predictions for the decay channels, even using just the leading-order results. For example, note from Table 1 that the $K = 0(1) N_{1/2}$ resonance couples only to $\eta(\pi)$. Indeed, the $N(1535)$ resonance decays to ηN 30–55% of the time despite lying barely above that threshold, while the $N(1650)$ decays to ηN only 3–10% of the time despite having much more comparable phase space to πN and ηN . This pattern clearly suggests that the π -phobic $N(1535)$ should be identified with $K=0$ and the η -phobic $N(1650)$ with $K=1$, the first fully field theory-based explanation for these phenomenological facts.

4. Configuration Mixing

As mentioned above, one does not expect quark-shell baryon states with a fixed number of excited quarks to be eigenstates of the full QCD Hamiltonian. Rather, configuration mixing likely clouds the situation. Consider, for example, the expectation that baryon resonances have generically broad $[O(N_c^0)]$ widths. One may ask whether some states escape this restriction and turn out to be narrow in the large N_c limit. Indeed, some of the first work⁵ on excited baryons combined large N_c consistency conditions and a quark description of excited baryon states to predict that baryons in the **70**-analogue have widths of $O(1/N_c)$, while states in an excited negative-parity spin-flavor symmetric multiplet (**56'**) have $O(N_c^0)$ widths.

In fact there arise, even in the quark-shell picture, operators inducing configuration mixing between these multiplets.¹⁴ The spin-orbit and spin-flavor tensor operators (respectively ℓs and $\ell^{(2)} g G_c$ in the notation of Refs. 6, 7, 30), which appear at $O(N_c^0)$ and are responsible for splitting the eigenvalues m_0 , m_1 , and m_2 , give nonvanishing transition matrix elements between the **70** and **56'**. Since states in the latter multiplet are broad, configuration mixing forces at least some states in the former multiplet to be broad as well. One concludes that the possible existence of any excited

baryon state narrow in the large N_c limit requires a fortuitous absence of significant configuration mixing.

5. Pentaquarks

The possible existence of a narrow isosinglet, strangeness +1 (and therefore exotic) baryon state $\Theta^+(1540)$, claimed to be observed by numerous experimental groups (but not seen by several others), remains an issue of great dispute. Although the jury remains out on this important question, one may nevertheless use the large N_c method described above to determine the quantum numbers of its degenerate partners.¹⁵ For example, if one imposes the theoretical prejudice $J_\Theta = \frac{1}{2}$, then there must also be pentaquark states with $I=1$, $J=\frac{1}{2}, \frac{3}{2}$ and $I=2$, $J=\frac{3}{2}, \frac{5}{2}$, with masses and widths equal that of the Θ^+ , up to $O(1/N_c)$ corrections.

The large N_c analogue of the “pentaquark” actually carries the quantum numbers of N_c+2 quarks, consisting of $(N_c+1)/2$ spin-singlet, isosinglet ud pairs and an \bar{s} quark. The quark operator picture, for example, shows the partner states we predict to belong to SU(3) multiplets **27** ($I=1$) and **35** ($I=2$).³¹ However, the existence of partners does not depend upon any particular picture for the resonance or any assumptions regarding configuration mixing. Since the generic width for such baryon resonances remains $O(N_c^0)$, the surprisingly small reported width (<10 MeV) does not appear to be explicable by large N_c considerations alone, but may be a convergence of small phase space and a small nonexotic-exotic-pion coupling.

6. $1/N_c$ Corrections

All the results exhibited thus far hold at the leading nontrivial order (N_c^0) in the $1/N_c$ expansion. We saw in Sec. 3 that $1/N_c$ corrections are essential not only to explain the sizes of effects apparent in the data, but in the very enumeration of physical states. Clearly, if this analysis is to carry real phenomenological weight, one must demonstrate a clear path to characterize $1/N_c$ corrections to the scattering amplitudes. Fortunately, such a generalization is possible: As discussed in Sec. 2, the constraints on scattering amplitudes obtained from the large N_c limit are equivalent to the t -channel requirement $I_t = J_t$. In fact, Refs. 27 showed not only that the large N_c limit imposes this constraint, but also that exchanges with $|I_t - J_t| = n$ are suppressed by a relative factor $1/N_c^n$.

This result permits one to obtain relations for the scattering amplitudes

incorporating all effects up to and including $O(1/N_c)$:

$$\begin{aligned}
S_{LL'RR'I_sJ_s} = & \sum_{\mathcal{J}} \begin{bmatrix} 1 & R' & I_s \\ R & 1 & I_t = \mathcal{J} \end{bmatrix} \begin{bmatrix} L' & R' & J_s \\ R & L & J_t = \mathcal{J} \end{bmatrix} s_{\mathcal{J}LL'}^t \\
& - \frac{1}{N_c} \sum_{\mathcal{J}} \begin{bmatrix} 1 & R' & I_s \\ R & 1 & I_t = \mathcal{J} \end{bmatrix} \begin{bmatrix} L' & R' & J_s \\ R & L & J_t = \mathcal{J} + 1 \end{bmatrix} s_{\mathcal{J}LL'}^{t(+)} \\
& - \frac{1}{N_c} \sum_{\mathcal{J}} \begin{bmatrix} 1 & R' & I_s \\ R & 1 & I_t = \mathcal{J} \end{bmatrix} \begin{bmatrix} L' & R' & J_s \\ R & L & J_t = \mathcal{J} - 1 \end{bmatrix} s_{\mathcal{J}LL'}^{t(-)} + O(\frac{1}{N_c^2}), (4)
\end{aligned}$$

One obtains this expression by first rewriting s -channel expressions such as Eqs. (1)–(2) in terms of t -channel amplitudes. The $6j$ symbols in this case contain I_t and J_t as arguments (which for the leading term are equal). One then introduces¹⁶ new $O(1/N_c)$ -suppressed amplitudes $s^{t(\pm)}$, for which $J_t - I_t = \pm 1$. The square-bracketed $6j$ symbols in Eq. (4) differ from the usual ones only through normalization factors, and in particular obey the same triangle rules.

Relations between observable amplitudes that incorporate the larger set s^t , $s^{t(+)}$, and $s^{t(-)}$ are expected to be a factor of $N_c = 3$ better than those merely including the leading $O(N_c^0)$ results. Indeed, this is dramatically evident in $\pi N \rightarrow \pi \Delta$, where sufficient numbers of amplitudes are measured (Fig. 1). For example, (c) and (d) in the first four insets give the imaginary and real parts, respectively, of partial wave data for SD_{31} (\circ) and $(1/\sqrt{5})DS_{13}$ (\square), which are equal up to $O(1/N_c)$ corrections; in (c) and (d) of the second four insets, the \circ points again are SD_{31} data, while \diamond represent $-\sqrt{2}DS_{33}$, and by Eq. (4) these are equal up to $O(1/N_c^2)$ corrections.

7. Pion Photoproduction

Meson-baryon scattering is not the only process that can be considered in the soliton-inspired picture. As long as one knows the isospin and spin quantum numbers of the field coupling to the baryon along with the corresponding N_c power suppression of each coupling, one may carry out precisely the same sort of analysis as described above.

The processes we have in mind are those involving real or virtual photons (photoproduction,¹⁸ electroproduction, real or virtual Compton scattering). One minor complication is that the electromagnetic interaction breaks isospin, in that the photon is a mixture of isoscalar ($I = 0$) and isovector ($I = 1$) sources. The former is suppressed by a factor $1/N_c$ com-

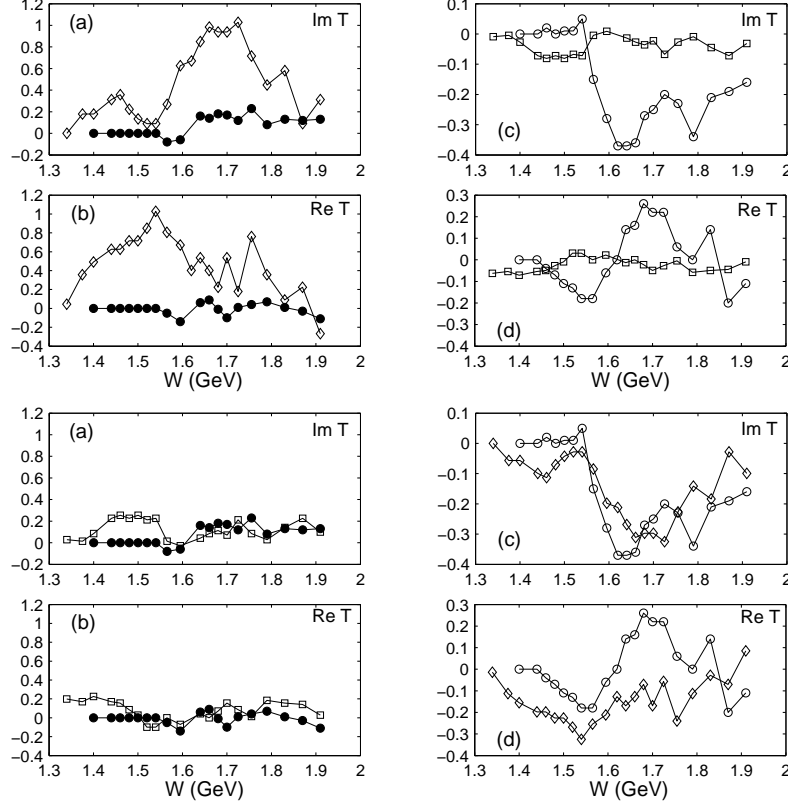


Figure 1. Real and imaginary parts of $\pi N \rightarrow \pi \Delta$ scattering amplitudes. The first four insets give two particular partial waves equal to leading order [hence indicating the size of $O(1/N_c)$ corrections]. The second four insets give two particular linear combinations of the same data good to $O(1/N_c^2)$.

pared to the latter since baryon couplings carrying both a spin index (coupling to the photon polarization vector) and an isospin index are larger than those carrying just a spin index by a factor N_c .³²

Moreover, electromagnetic processes are typically parametrized in terms of multipole amplitudes, which combine the intrinsic photon spin with its relative orbital angular momentum; in fact, this is very convenient, because then the photon can be treated effectively as a spinless field whose effective orbital angular momentum is the order of the multipole. Note that this makes processes with virtual photons just as simple as those with real photons, even though the former can carry not only spin-1 but spin-0 ampli-

tudes as well. With these caveats in mind, carrying out an analysis of pion photoproduction amplitudes, including $1/N_c$ corrections (leading plus sub-leading $I=1$ amplitudes and leading $I=0$ amplitudes), is straightforward.¹⁸

For example, a relationship receiving only $O(1/N_c^2)$ corrections reads

$$M_{L,L,-}^{m,p(\pi^+)n} = M_{L,L,-}^{m,n(\pi^-)p} - \left(\frac{L+1}{L} \right) \left[M_{L,L,+}^{m,p(\pi^+)n} - M_{L,L,+}^{m,n(\pi^-)p} \right], \quad (5)$$

where the superscript m means magnetic multipoles, $N(\pi^a)N'$ means the process $N\gamma \rightarrow N'\pi^a$, and the subscripts L, L, \pm mean that an electromagnetic multipole of order L creates a pion in the L^{th} partial wave, with total $J=L\pm\frac{1}{2}$. Including just the first term on the right-hand side (r.h.s.) gives a relation valid up to $O(1/N_c)$ corrections, and the quality of both this relation and its extension to next-to-leading order may be assessed.

A sample result appears in Fig. 2, where the left-hand side (l.h.s.) is a solid line, the $O(1/N_c)$ result is dotted, and the $O(1/N_c^2)$ is dashed. While the agreement at first glance may not seem impressive, some very heartening features may be discerned. First, the agreement in the region below the appearance of resonances is quite good, and indeed improves at $O(1/N_c^2)$. Second, unlike the solid line [containing $D_{13}(1520)$], the dotted line gives no hint of a resonance but the dashed line does [$D_{15}(1675)$]; and the fact that their positions do not precisely match should not alarm us, as one expects them to differ by an amount of $O(\Lambda_{\text{QCD}}/N_c) \sim 100$ MeV. One may in fact use the helicity amplitudes compiled¹ for these two resonances and relate them directly to the amplitudes appearing in Eq. (5). In order to obtain dimensionless and scale-independent results, one divides the linear combination of helicity amplitudes corresponding to Eq. (5) by the same expression with all signs made positive. The $O(1/N_c)$ and $O(1/N_c^2)$ combinations give¹⁸ -0.38 ± 0.06 and -0.13 ± 0.06 , respectively, showing that the $1/N_c$ expansion works beautifully—even better than one might expect.

8. Conclusions: The Way Forward

There now exist reliable and convincing calculational techniques using the $1/N_c$ expansion of QCD that handle not only long-lived ground-state baryons, but also unstable baryon resonances and the scattering amplitudes in which they appear. The approach, originally noted in chiral soliton models but eventually shown to be a true consequence of large N_c QCD, is found to have phenomenological consequences [such as the large η branching fraction of the $N(1535)$] that compare favorably with real data.

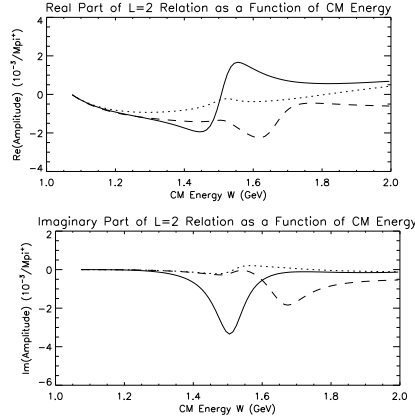


Figure 2. Comparison of Eq. (5) with data for $L = 2$. The solid line is the l.h.s., the dotted line is the first term on the r.h.s., and the dashed line is all r.h.s. terms.

The first steps of obtaining $1/N_c$ corrections to the leading-order results, absolutely essential to make comparisons with the full data set, are complete. The measured scattering amplitudes appear to obey the constraints placed by these corrections, and more work along these lines is forthcoming. For example, the means by which the spurious extra resonances of $N_c > 3$ decouple as one takes the limit $N_c \rightarrow 3$ is crucial and not yet understood.

The explicit results presented here, as mentioned in Sec. 2, have used only relations among states of fixed strangeness. Moving beyond this limitation means using flavor $SU(3)$ group theory, which is rather more complicated than isospin $SU(2)$ group theory. Nevertheless, this is merely a technical complication, and existing work shows that it can be overcome.^{7,17}

At the time of this writing, all of the essential tools appear to be in place to commence a full-scale analysis of baryon scattering and resonance parameters. One may envision a sort of resonance calculation factory, which I have previously dubbed *Baryons IN_C*.¹⁹ Given sufficient time and researchers, the whole baryon resonance spectrum can be disentangled using a solid, field-theoretical approach based upon a well-defined limit of QCD.

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